11/2 Twoples

- Will start writing after learning localization/equivalences.
- This past week: read Joyal's paper "Quasicats & Kan cpxes"
  Dwyer-Kan re: simplicial localization. (in HTT?)

underlying appears in [HTT, A.2], but he doesn't describe simp. loc'n.

Def. $\mathcal{O}$: a set. $\rightsquigarrow$ $\boxed{\mathcal{O}\text{-Cat}}$ is a cat.

- ob $= \{$cats C w/ $ob(C) = \mathcal{O}\}$
- mor $= \{F : C \to D$ s.t. $Fx = x$ $\forall x \in ob(C) = ob(D) = \mathcal{O}\}$.

Def. $\boxed{\text{free product}}$ of cats $\coprod C_i$ ...

Def. $\boxed{s\mathcal{O}\text{-Cat}} = \{$simplicial objects in $\mathcal{O}$-Cat$\} = \mathrm{Fun}(\Delta^{op}, \mathcal{O}\text{-Cat})$

Def. A cat is $\boxed{\text{free}}$ if $\exists$ a set of "generating morphisms" $S \subseteq \mathrm{Hom}(C) \setminus \{id's\}$
s.t. any (non-id) in C is a comp. of maps in S.

Prop: free prod's of free cats are free.

Prop: every free cat is a free product of cats w/ only one generator. ??

(Pf?) Let C be a free $\mathcal{O}$-cat. ie. $\exists S \subseteq \mathrm{Hom}(C)$ s.t. any $x \longrightarrow y$ ($\bullet \to \bullet \cdots$)

Claim: then $C = \coprod_S [1]_S$?

where $[1]_S = \{\bullet \xrightarrow{f} \bullet\}$ where $f \in S$.

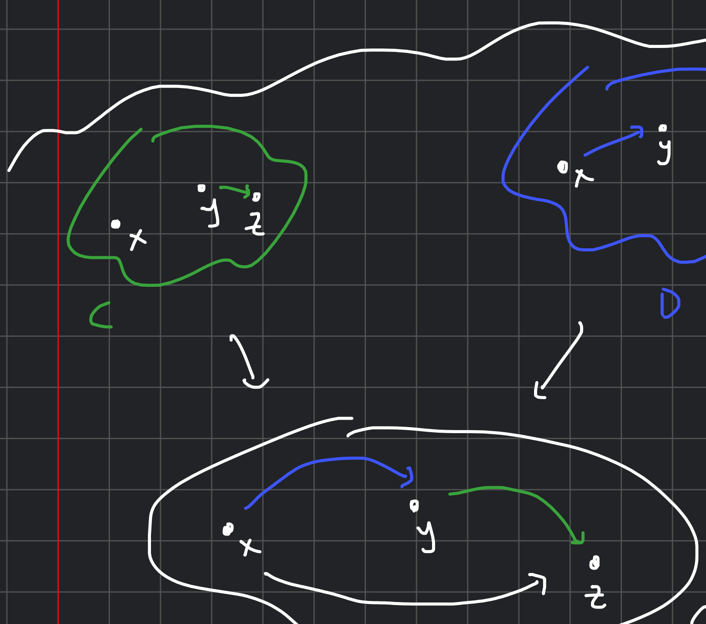
$\mathcal{O} = ob(C) = \coprod_S ob([1]_S)$

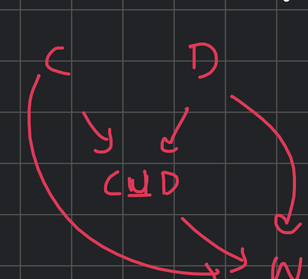
Pick $x, y \in C$ ... Then $\mathrm{Hom}(x,y) =$

The $\coprod$ in $\mathcal{O}$-Cat is not the same as in Cat.

Cat	$\mathcal{O}$-Cat
C, D	C, D
	$ob(C) = \mathcal{O}$, $ob(D) = \mathcal{O}$
$C \sqcup D$	
$C \amalg D$	$\mathcal{O} \amalg \mathcal{O}$ ✗
	$ob(C \amalg D) = \mathcal{O}$

Claim: $\mathrm{Hom}_{C \amalg D}(x,y) \cong \{$compositions $\bullet \xrightarrow{C} \bullet \xrightarrow{D} \bullet \xrightarrow{C} \cdots \xrightarrow{D} \bullet\}$





$C \amalg D$ — ob $= \mathcal{O}$
— $\mathrm{Hom}_{C \amalg D}(x,y) =$ strings

simplicial localization $M \mapsto L^{DK}M$ ...

Hammock localization (a lot easier) — used in Mazel-Gee.
$M \mapsto L^H M$

$L^{DK}M \longleftrightarrow L^H M$ are equivalent (in a suitable sense)

$$F_* C \xrightarrow{\varphi} \overline{C}$$

$$\begin{matrix} \mathcal{O} & \xrightarrow{id} & \mathcal{O} \\ \| & & \| \\ ob(F_* C) & & ob(\overline{C}) \end{matrix}$$

$$F_n C \xrightarrow[\varphi^n]{} \overline{C}_n = C$$

const. s.cat.
$\overline{C} \in s\mathcal{O}\text{-}Cat$

$\underset{\overline{C}}{Hom}(x,y)_\bullet \in sSet$

$\underset{\overline{C}}{Hom}(x,y)_n = \underset{C}{Hom}(x,y)$

& face/deg. maps are id's.

X

"Zig zags" ...

eg. We say two top. spaces $X, Y \in Top$ are weak htpy equiv. if connected by a zig-zag of weak htpy equiv's.

$$X \xrightarrow{\sim} A_1 \xleftarrow{\sim} A_2 \xrightarrow{\sim} A_3 \xleftarrow{\sim} \cdots \xrightarrow{\sim} A_n \leftarrow Y$$

in $hTop$.

$$X \xrightarrow[\sim]{} Y.$$

X

A weak eq. of scats is a functor inducing isos of all htpy gps of all Hom-ssets.

Two scats are weak htpy. equiv. if they're connected by a zig-zag...

eg.

$$L^H C \xleftarrow{\sim} \operatorname{diag} L^H F_* C \xrightarrow{\sim} F_* C[F_* W^{-1}] = LC$$

ie. $L^H C \underset{w.eq.}{\sim} LC$ ie. $L^H C \cong LC$ in $h(sCat)$

First D-K paper

X

an eg. of a Quillen equiv. $\rightsquigarrow$ ∞-equiv.?

↳ topological spaces & ssets?